



# Examiners' Report Principal Examiner Feedback

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Pearson Edexcel International Advanced Level  
In Pure Mathematics (WMA14) Paper 01

# **WMA14 OCTOBER 2023**

## **Examiners' Report**

### **General**

This paper proved to be a good test of candidates' ability on the WMA14 content and plenty of opportunity was provided for them to demonstrate what they had learnt. Marks were available to candidates of all abilities and the questions that proved to be the most challenging were 4, 5(b), 6(d) and 8(d)(i). Of these, question 4 appeared to cause the most problems, with many candidates not knowing how to start the proof by contradiction. It is worth commenting on the relatively large number of candidates who ignored (or possibly didn't see) the requirement to use integration by substitution with question 3(ii) and proceeded to use parts or partial fractions with no substitution seen. Presentation was generally good and candidates often showed sufficient working to make their methods clear.

### **Question 1**

Part (a) was generally well answered, with many candidates gaining all 4 marks. If marks were lost, this tended to be due to failing to extract  $2^{-2}$  properly, leading to  $2(1 + \dots)$  or even  $16(1 + \dots)$ . The structure of the binomial expression was usually correct, but some candidates used  $x$  or  $-5x$  (or sometimes a mix) instead of  $-\frac{5x}{2}$  in their expansions. Some lost the final mark due to sign errors or numerical slips in simplifying terms, but fully correct answers were common. In a small number of cases a direct expansion was attempted, often unsuccessfully, with a range of errors including the attempt to evaluate  ${}^nC_r$  with  $n < 0$ .

Part (b) was poorly answered, with many candidates unaware what was required or unable to use the expression  $|x| < 1$  (given in the formula booklet) to manipulate  $\left| -\frac{5x}{2} \right| < 1$  into  $|x| < \frac{2}{5}$ . Wrong answers included  $x < \frac{2}{5}$ ,  $|x| < \frac{5}{2}$ ,  $|x| > \frac{2}{5}$  and  $-\frac{1}{5} < x < \frac{1}{5}$ .

### **Question 2**

This was a reasonably straightforward question that most candidates had a suitable approach for. There were very few candidates scoring no marks, but errors were common.

In part (a), most candidates correctly interpreted the given information and stated or used the fact that  $\frac{dS}{dt} = 4$ . There was pleasingly little confusion with variables with very few confusing this with  $\frac{dS}{dx}$  although a significant number of candidates used "A" rather than "S" but this was condoned. A common source of error was in formulating an expression for the surface area of the cube with  $S = x^2$  seen frequently.  $S = 4x^2$  and  $S = 8x^2$  were also seen occasionally. Most candidates went on to use a correct formulation of the chain rule with a small number differentiating their expression for  $S$  implicitly.

The vast majority of candidates had a valid approach for part (b), starting with  $V = x^3$  and using the chain rule correctly. Errors mainly followed from an incorrect expression for  $\frac{dx}{dt}$  (usually from an incorrect expression for  $S$ ) from part (a). A surprising number of candidates left their final answer as  $\frac{dV}{dt} = x$ .

### Question 3

Part (i) was well answered, although a few candidates ignored the limits and only evaluated the indefinite integral (for a maximum of 3 out of 5 marks). Some candidates integrated by parts ‘the wrong way round’, introducing higher powers of  $x$  and making no progress. There were some mistakes with factors of 2 instead of  $\frac{1}{2}$  and also with signs, due to not dealing correctly with brackets during the double application of integration by parts. For those candidates who did use the limits, the assumption that  $e^0 = 0$  was common. A few gave their answer as a decimal rather than in the exact form required.

There was a small number of candidates who chose to use the “DI table” method for integration by parts and such attempts had various degrees of success. Of those who opted for this approach, any errors were often with incorrect coefficients when integrating the exponential term but generally the overall method was sound.

Part (ii) was also relatively well answered. Most candidates used the substitution  $u = 2x - 1$  and proceeded to gain most of the marks. Alternative substitutions were rarer and often unsuccessful. Errors seen tended to be due to slips, e.g. with signs and then rearranging  $u = 2x - 1$ . The required integration was often successfully performed, though occasionally limits for  $x$  were substituted into  $\ln u - \frac{1}{u}$ . Some candidates ignored the instruction to use substitution and instead used parts or partial fractions, gaining a maximum of 4 out of the 7 marks available.

### Question 4

Many candidates found part (a) challenging. Many began with an incorrect assumption though they did then go on to attempt to establish a contradiction so showed some understanding of the nature of these types of proof. A significant number of candidates showed very little appreciation of a proof by contradiction, using specific values of  $k$  in the expression and concluding that it must therefore be true, or attempting a purely algebraic proof without attempting to establish a contradiction at all. The most common incorrect assumption was to assume that  $k + \frac{9}{k} \geq 6$  for negative values of  $k$ . It was also common to see confusion with signs and for the assumption that there exists a value of  $k$  for which  $k + \frac{9}{k} \leq 6$ . Few candidates appreciated that they only needed to establish that there is one value of  $k$  for which  $k + \frac{9}{k} < 6$  with many stating their assumption as “assume that for all positive  $k$ ,  $k + \frac{9}{k} < 6$ ” though this was a subtlety that was condoned.

Having obtained a correct starting point the algebraic step was then fairly straightforward and many candidates scored the M1A1 for this step. Obtaining  $(k-3)^2 < 0$  was the most common approach though squaring both sides and use of a sketch or the discriminant were also seen. Candidates using the discriminant generally did not give any explanation for what this showed.

Most candidates that established that  $(k-3)^2 < 0$  then realised that this gave rise to a contradiction though there was often a lack of precision in their reasoning with some candidates stating that  $(k-3)^2 > 0$  and this was more common with statements given in words.

In part (b), many candidates did not appreciate the demand of the question here and embarked on another algebraic proof. Those that recognised that all that was needed was to substitute in a negative value usually scored the mark. A small number successfully constructed an appropriate argument with  $k$  as a non-specified general negative number. The most common error amongst those who realised that a counterexample was needed was to choose  $k = 0$ .

### Question 5

In general, this question was done well and most candidates scored highly on it. Only occasionally were methods other than implicit differentiation attempted and approaches using partial differentiation were seen very occasionally.

In part (a) there were a number of slips with coefficients of derivatives and a significant minority of candidates forgot to differentiate  $k$  but the actual implicit differentiation and product rule was generally accurately done. Some candidates, however, failed to treat  $8xy$  as a product. There were fewer candidates than expected starting with  $\frac{dy}{dx} = \dots$ . On the whole, the manipulation and factorisation of the derivative was well done and the majority of candidates achieved full marks.

Many candidates were less confident with part (b) of the question and equated their derivative to zero. Others equated their derivative to  $-1$  but failed to appreciate the role  $y = x$  played in defining the value for the gradient of the curve, so were unable to solve for  $x$  or  $y$ . These candidates generally gave up part way through. Those who did realise what method was required were often able to gain the allocated marks easily. A few candidates, who incorrectly left  $k$  in their derivative, in this part found  $k$  in terms of  $x$  using  $\frac{dy}{dx} = -1$  and  $y = x$  and then attempted to solve this and the equation of the curve simultaneously to find a value of  $x$ .

## Question 6

This question was structured in such a way to allow candidates to access at least some of the parts.

In part (a), whilst most candidates gave the correct point it was surprising to note the number who did not. There was a significant number of candidates giving  $(0, 0)$  as the coordinates of  $P$  and a large number who, although they gave the correct point, went through a very longwinded approach of equating the two lines to establish that  $\lambda = \mu = 0$  and substitute that back into the equation of the line rather than simply stating the coordinates as in the demand of the question.

In part (b), the majority of candidates knew how to use the scalar product to find  $\cos\theta$  and used the two direction vectors to do so, achieving the full 3 marks. A small number of candidates used

$\mathbf{a} = \mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix}$  or used values of  $\lambda$  and  $\mu$  to find the coordinates of points on each line and used those

to find  $\cos\theta$ . A small number of candidates did not simplify their answer.

Many candidates had a good go at part (c) although some candidates used  $\lambda = 6$  to find the coordinates of  $Q$  and used the length of  $OQ$  in a formula for the area of the triangle. It was common to see an inefficient approach where candidates found the coordinates of  $Q$  as a first step before then finding the vector  $PQ$ , not realising they could go straight to  $\lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ . Having found  $PQ$  there

were subsequently some errors in finding the area of the triangle such as:

- omission of the  $\frac{1}{2}$
- an attempt to find  $RQ$  as the second length
- the use of  $\frac{1}{2} \times 18 \times 2 \sin \theta$
- the use of  $\frac{1}{2} \times 18 \times 18 \cos \theta$

Candidates using a sketch diagram were usually successful with part (d) of this question and there were a good number of attempts at this part with mixed success. Candidates who had found the length of  $PQ$  in part (c) generally went on to use a correct method to find at least  $\mu = 2$ . However, it was common to see candidates equating the vectors  $PQ$  and  $PR$  and then using the components to find values for  $\mu$ .

## Question 7

The quality of solutions to this question was extremely variable with some parts producing consistently high scores.

In part (a) most candidates substituted  $x = 3$  and  $t = 0$  correctly and found  $k$  with all necessary working included. Some substituted  $x = 3000$  and found  $k = 1800$ , then divided by 1000 to achieve the required value of  $k$ , losing a mark. Very few lost marks due to lack of working.

In part (b) a good number of candidates managed to achieve 4.05 but did not relate the value to the context of the question and failed to follow up with an answer of 4.05 thousands or 4050. Some had little idea how to find limiting values and failed to score.

The required partial fractions in part (c) were usually correct, with most candidates scoring all 3 marks. Just a few made numerical errors which led to incorrect values of  $A$  or  $B$ . The partial fractions structure and method was well known.

Weaker candidates often stopped after the partial fractions and could not see how to apply their answers to the differential equation in part (d). Those who attempted this part tended to do well and separated variables correctly, although some rearranged the 3 to be a coefficient of  $t$  and then used their partial fractions incorrectly. Once separated, the integration was generally very good with most candidates recognising the  $\ln$  integrals, usually with correct coefficients. The main problem at this point was the lack of a constant of integration, which then meant that no further marks were available. Many candidates rearranged and dealt with the logs correctly but were let down by their failure to consider the integration constant. Many, however, did achieve the given answer with good algebra and log work.

Many candidates who attempted part (e) were successful, and some skipped to this part whilst missing out previous parts of the question. Others did not attempt it, despite all necessary information for its completion being given in the question. Again, there were those who achieved 4.5 but did not relate it back to the context of the question, so did not give the answer 4500.

## Question 8

There was some evidence to suggest that candidates were running out of time to complete this question.

In part (a), most candidates obtained  $3\pi$  but a significant number of candidates left this part of the question unanswered or gave an answer of  $\frac{\pi}{2}$ .

The vast majority of candidates understood the process of finding  $\frac{dy}{dx}$  in part (b) and obtained the correct form for  $\frac{dy}{dt}$  and usually  $\frac{dx}{dt}$  before correctly using their derivatives to form an expression for  $\frac{dy}{dx}$ . There were occasional sign errors in  $\frac{dy}{dt}$  and some incorrect methods for finding  $\frac{dx}{dt}$  with  $6 - 3\cos 2t$  and  $6 - 6\sin 2t \cos 2t$  being seen. Unfortunately, these errors made it impossible to reach the correct form for  $\frac{dy}{dx}$ . Having obtained a correct expression for  $\frac{dy}{dx}$  most candidates were able to go on to obtain an expression of the required form using a correct trigonometric identity – the most common error being omission of the 2 from  $\cos 2t = 1 - 2\sin^2 t$ .

Part (c) of this question proved more challenging with many candidates leaving it out and others simply giving  $\sqrt{2}$  (the y-coordinate of  $P$ ) as the y-coordinate of  $N$ . Many candidates who had a valid strategy obtained full marks though errors in  $\frac{dy}{dx}$  or use of inexact values for the coordinates of  $P$  lead to a loss of marks. Some made algebraic errors having found a correct equation, including the  $\pi$  disappearing as the surds were manipulated. A small number did not evaluate the trigonometric expressions.

In (d)(i), a good number of candidates knew that they needed to find  $y^2 \frac{dx}{dt}$  and it was common for candidates to obtain the first 2 marks. Unfortunately, many candidates then struggled to obtain an integrand of the correct form and did not score any further marks. Some candidates attempted  $\int y^2 dt$  or quoted the correct formula but then  $dx$  became  $dt$  without consideration of  $\frac{dx}{dt}$ .

Some otherwise good attempts lost the final mark due to an incorrect upper limit (usually  $3\pi$ ) or, very occasionally, loss of the  $\pi$  from  $\int \pi y^2 \frac{dx}{dt} dt$ .

Unfortunately, in (d)(ii), many candidates who struggled to obtain an integrand of the required form did not attempt this part of the question. However, it is worth noting that the method mark was available for correct integration either in terms of  $\beta$  or with any value they might have obtained.